Statistical equilibrium in a simulated double auction

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Abstract - We study the properties of the stationary distribution of shares, cash and wealth for an artificial stock market in which zero-intelligence agents trade via a continuous double auction. We compare these properties with the results recently published by Silver et al. using a Walrasian auction.

1 Introduction

Describe results of Silver et al.

2 Simulations

Our simulations feature a continuous double-auction implemented on the *Nat-Lab* platform [sp, MLS06], which can simulate continuous time asynchronous processes. We kept the market mechanism (the *rules of the game*) as simple as possible, while retaining the concept of continuous double-auction, essential to understand the price formation dynamics.

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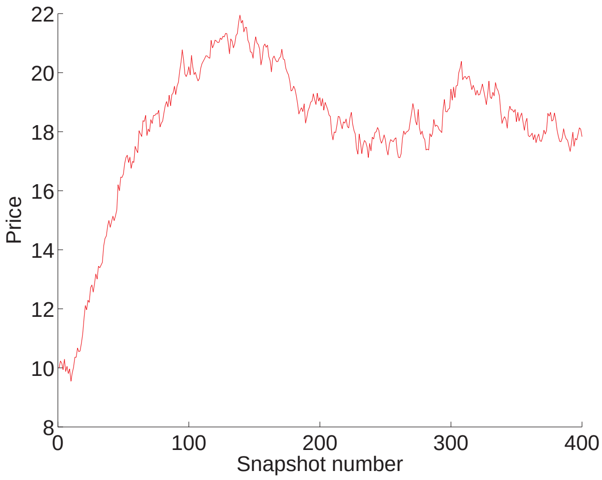


Figure 1: Price as a function of time. Snapshots were taken every 10 000 virtual seconds, during which each of the 1000 agents submitted an average of 10 orders

In our artificial stock market, zero-intelligence random agents submit, asynchronously and at any time, limit or market orders to a single public book. Orders are sorted by price and then by time, as on the *NYSE* for instance [HSSD93]. Every agent acts as a simple trader – we do not include brokers or market makers. In this simple setup, agents balance their portfolio between a risky asset (a stock not distributing dividends) and a riskless one (equivalent to cash).

Our computer simulation figures a closed artificial market, with no creation of stocks, no distribution of dividends and no interest rate associated with cash. In the specific run presently reported, there were $N = 1000$ agents interacting exclusively through the market and initially endowed with $S_0 = 100$ shares and $C_0 = 2000$ units of cash. The initial stock price was set to the out-of-equilibrium value $P_0 = 10$. After a transient, the price converges towards the equilibrium price $P^* = \frac{C_0}{S_0} = 20$, as can be seen on Figure 1.

During the evolution of the system, agents exchange cash and shares through the limit order book, leading to a random distribution of the two variables. Wealth is defined as

$$W(t) = C(t) + p(t)S(t), \tag{1}$$

where $C(t)$, $S(t)$ and $p(t)$ stand for cash, shares and price at time t respectively.

3 Theory

Describe Markov Webs Describe the state spaces Y and Z for the Markov Chain

	Shares	Cash	Wealth
Variance	0.409 ± 0.003 [1]	0.369 ± 0.002 [1]	0.019 ± 0.003 [0.5]
Skewness	0.266 ± 0.012 [2]	0.142 ± 0.012 [2]	0.516 ± 0.003 [$\sqrt{2}$]
Kurtosis	2.130 ± 0.021 [6]	2.080 ± 0.018 [6]	2.560 ± 0.006 [3]

Table 1: Mean and three standard errors of the moments studied (variance, skewness and kurtosis), computed over the last 100 snapshots. The theoretical value is given into square brackets, following the format $Mean \pm 3.StdError\ [Theory]$

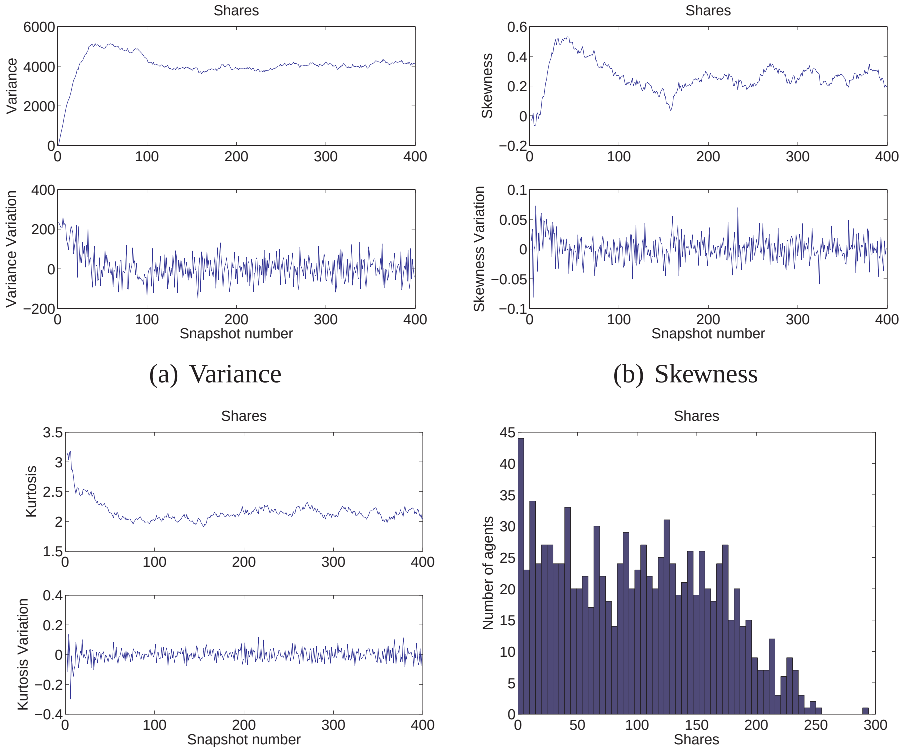
In the double auction market described above, the state space is given by all the possible allocations of shares and cash among the agents. However, these states do not always correspond to the same total wealth as the price of shares endogenously fluctuates, as can be seen on Figure 1. Recently, Silver et al. studied Walrasian allocation of two goods with uniformly distributed random preferences, A and B, in a framework that has the same state space as ours as long as cash is interpreted as good A and shares are interpreted as good B [SST02]. Using a heuristic maximum entropy argument, Silver et al. show that the normalized equilibrium distribution for shares and cash in such a Walrasian market is given by a $Gamma(1, 1)$, whereas for wealth, they derive a $Gamma(2, 2)$ distribution. In the next section, we study the properties of the stationary distributions resulting from the double auction mechanism with zero-intelligence agents. It is important to remark that our agents issue orders according to a target price that is normally distributed around the last market price.

4 Results

Since we are particularly interested in assessing whether we reach a stationary state that could coincide with statistical equilibrium for the closed market above described, we study the time evolution of the variance, skewness, and kurtosis of the agent distributions of shares, cash and wealth. They are given in Figures 2, 3 and 4 for typical simulation run.

It appears that the system has reached an equilibrium state in which the distributions of shares, cash and wealth are stationary. We can now compare the moments of these distributions with those predicted by [SST02].

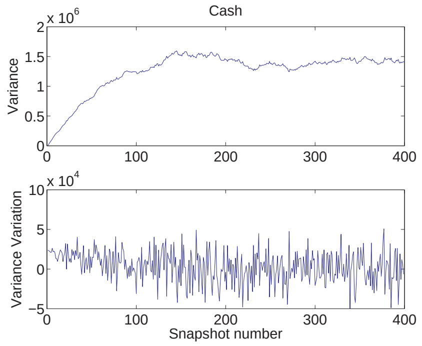
From the preliminary analysis presented in Table 1, we reach the conclusion that the stationary distributions do not coincide with those predicted by [SST02]. We are now exploring the origin of this discrepancy, to be related to the market microstructure or the agents' behaviour.



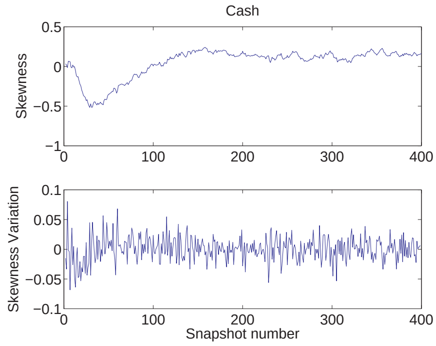
(a) Variance (b) Skewness

(c) Kurtosis (d) Final distribution

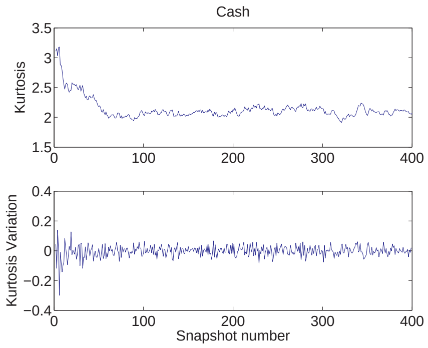
Figure 2: Time evolution for the moments of the distribution of shares. Top left: variance and its variation; Top right: skweness and its variation; Bottom left: kurtosis and its variation; Bottom right: distribution of shares after 400 snapshots.



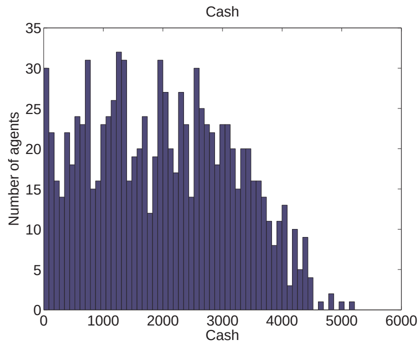
(a) Variance



(b) Skewness

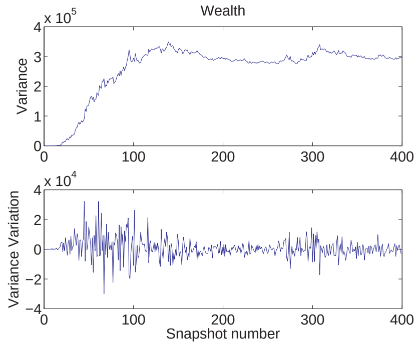


(c) Kurtosis

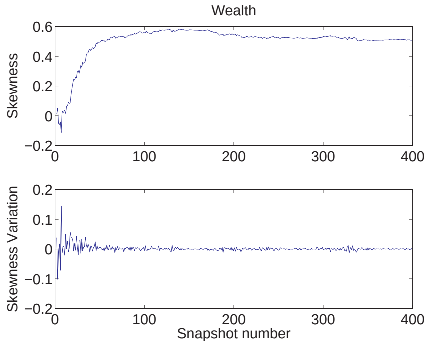


(d) Final distribution

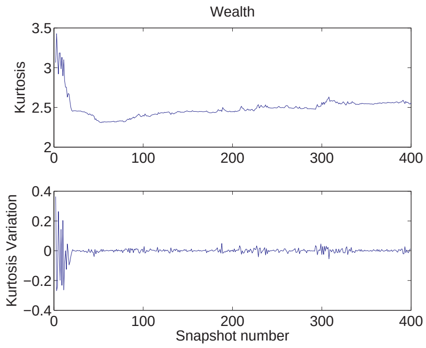
Figure 3: Time evolution for the moments of the distribution of cash. Top left: variance and its variation; Top right: skweness and its variation; Bottom left: kurtosis and its variation; Bottom right: distribution of shares after 400 snapshots.



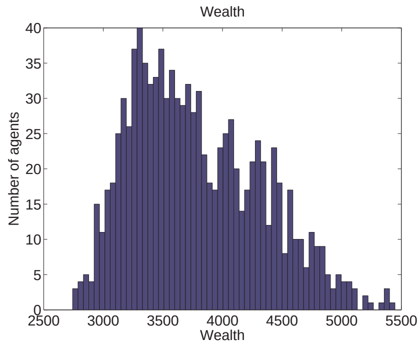
(a) Variance



(b) Skewness



(c) Kurtosis



(d) Final distribution

Figure 4: Time evolution for the moments of the distribution of wealth. Top left: variance and its variation; Top right: skweness and its variation; Bottom left: kurtosis and its variation; Bottom right: distribution of shares after 400 snapshots.

5 Discussion

We analysed a small system and extrapolate from it

References

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